ANALYSIS OF THREE-PHASE INDUCTION MOTORS UNDER CONDITIONS OF TECHNICAL AND ECONOMICAL UNCERTAINTY AIMING ENERGY CONSERVATION

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Abstract – The developed methodology allows to check the technical and economical feasibility for substitution of oversized three-phase induction motors aiming energy conservation. The uncertainties inherent to the measuring process of performance characteristics of motors to be replaced are considered. Its adequate use allows to make decisions about an eventual substitution, reducing the present risks.

I. INTRODUCTION

The determination of operating characteristics in three-phase induction motors retains sheer importance for the evaluation of their performance in load driving, mainly from the energy conservation standpoint, since the motors make a meaningful share in the industrial consumption of energy.

Motors in partial load operation are usually found, causing low power factor and low efficiency, thus constituting a point of electrical energy waste. Therefore, in many ways, the replacement of one motor to another which offers more favourable operational characteristics is interesting.

Several methodologies [1-3] designed to obtain such characteristics in field have been developed, and with them, many discussions [4-7] about their accuracy and reliability in relation to the methodologies preconized in specific standards [8-11] for tests in laboratories, including comparison between the two last kinds.

One of the non-standardized methodologies is the linearization of the torque vs. speed curve. In this case, with the motor in its real working conditions, the shaft power (P_L) can be achieved, for a given load. Only the developed speed is employed as a basic measure.

This work improve the linearization method, incorporating uncertainties, for the scope of low risk (5%) in an eventual motor replacement.

After the technical analysis, an economic evaluation is carried out also under uncertainty conditions. These conditions are attached to the number of working hours, energy tariff, efficiency, useful life, investment, interest rates and residual value.

Both technical and economical analysis make use of the Monte Carlo Method for the treatment of random variables.

II. CONCEPTUAL ASPECTS

For the calculation of the working torque, the method employs a linearization of the torque curve in function to speed in three-phase induction motors.

As shown in Fig. 1, the section in the linearization curve is the one considered as an operating region, i.e., between the rated condition, defined by the rated speed (n_R) and the rated torque (T_R) coordinates, and the condition with free rotor. In this condition, the rotor speed is adopted to be the same as the revolving field (synchronous) speed (n_S) and thus, the developed torque in the motor shaft is null.

Making use of these points, the expression that defines the working torque (T_L) will be:

\[
T_L = T_R \cdot \frac{n_S - n_L}{n_S - n_R}
\]

where n_L is the motor working speed to be measured, and the rated torque (T_R) is calculated from the motor nameplate.

Fig. 1. Linearization process of the torque curve.
This way, the working power \((P_L)\) on the shaft can be calculated by:

\[
P_L = T_L \cdot n_L
\]  

(2)

III. TECHNICAL UNCERTAINTIES

Through this methodology, the technical uncertainties in the calculation of the working power of a three-phase induction motor show up due to an unaccuracy in the measurements, to variations that occur in the motor nameplate data (values obtained from tests performed in some samples) and to errors existing between the real curve and the proposed straight line.

In order to analyse these variations statistically, experimental data were used in a total of 20 three-phase induction motors, from different rated power and speeds found in [3].

Considering the rated nameplate power \((P_{R \text{ rated}})\) as a reference value, the speed associated to it, called real rated speed \((n_{R \text{ real}})\), has been obtained. In order to generalize the calculations, the speed was defined in per unit (pu), using each motor’s rated speed as a basis.

\[
\eta_{\text{pu}} = \frac{n_{\text{R real}}}{n_R}
\]  

(3)

Using (3), Table 1 can be built, and with these data, the average and standard deviations can now be evaluated.

<table>
<thead>
<tr>
<th>Motor</th>
<th>(n_{\text{R real}})</th>
<th>(n_R)</th>
<th>(\eta_{\text{pu}})</th>
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<td>0.99899</td>
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All distributions will be considered normal. In case of the error, its average is zero, for the absence of systematic errors in the instrument has been admitted.

Taking the technical uncertainties about the nameplate rated speed and working speed in account, the distribution of the linearized working torque is achieved.

\[
\bar{T}_L = T_R \cdot \frac{n_S - \bar{n}_L}{n_S - n_R}
\]  

(6)

Besides, we should consider the uncertainties introduced by the method of linearization into working torque calculation, because it is known to exist a difference between the real curve and the linearized curve. Fig. 2 shows a comparison between these curves for a 11 kW motor with rated torque of 67.8 Nm at 1740 rpm.

To obtain this error, the bulk data from experimented motors were taken, and the deviation in pu between the linearized torque and the actual torque (measured) was calculated for the various experimental speeds of each motor in the following way:

\[
\Delta T_{\text{pu}} = \frac{T_A - T_i}{T_R}
\]  

(7)

where \(T_A\) is the actual torque obtained by tests and \(T_i\) is the linearized torque.
The Monte Carlo Method consists in performing a high number of simulations, in which in each one, an assortment is done for the random variables involved. This assortment has to respect these variables distributions. For each simulation, the desired physical quantity is calculated (in case, $P_L$), and at the end, the average and standard deviation are calculated from the several values obtained.

Nowadays, such procedure is an easy achievement due to the computer evolution and its "languages" which incorporates routines for random numbers generation.

With the distribution of working power, it is possible to choose a motor that supports the load, admitting a maximum risk. This risk is the probability of being the load upper than a fixed value.

It is usual in engineering to define the risk of 5% as something acceptable. The chosen motor will carry a working power greater than the expected value (average) as it is illustrated in Fig. 3.

To obtain this 5% risk point, a proposed polynomiun will be used [12], which is an approximation to a normal pattern distribution.

\[
z = \frac{c_0 + c_1 \cdot t + c_2 \cdot t^2}{1 + d_1 \cdot t + d_2 \cdot t^2 + d_3 \cdot t^3}
\]

where

\[
t = \sqrt{\ln(1/Q^2)}
\]

Fixing $Q$, which in this case is 0.05 (5%), calculate $z$, and get $P_{L,95\%}$ as follows:

\[
P_{L,95\%} = \mu_{P_L} + z \cdot \sigma_{P_L}
\]

Where $\mu_{P_L}$ is the average and $\sigma_{P_L}$ is the standard deviation of the probability distribution associated to working power.
V. ECONOMICAL EVALUATION

In this work, the use of the Monte Carlo Method is also proposed for analysis of the economic feasibility of motor substitution, considering the existence of a group of associated uncertainties.

The investment on the new motor ($I_M$), the residual value of the motor in use ($V_{10}$), the tariff of electrical energy ($TE$) and the motor operation time per year ($H$) are variables that contain a high level of uncertainty. A practical way to quantify these variables is to employ a triangular distribution, presented in Fig. 4.

In this case, the most probable value (MPV), the lower value (LV) and the upper value (UV) are estimated.

As this technique has been developed, for normal distributions, an approximation is performed. Thus, assuming a confidence interval of 80%, it is possible to calculate the average ($\mu$) and the standard deviation ($\sigma$) as follows:

$$\mu = \frac{LV + MPV + UV}{4} \quad (13)$$

$$\sigma = \frac{UV - LV}{2.65} \quad (14)$$

In the case of the new motor's efficiency and the old motor's efficiency ($\eta_{new}$ and $\eta_{old}$), these values can be obtained from manufacture's curves and/or from tests.

There is an inherent uncertainty, of course. In the lack of further information, a deviation for the efficiencies around 1% and 2% can be assumed, basing oneself on standards prescriptions.

Finally, the last variable involved is the motor useful life. Based on data found in [7], we can have lower, upper and most probable values as shown in Fig. 5.

Fig. 5. Motor working life.

Using the triangular distribution, and proceeding the calculations described before, the average and the standard deviation for a normal approximated distribution are obtained. With the distributions of the variables involved in the economical calculations, a deeper analysis can be carried out.

In the cash flow adopted, $\bar{AB}$ is the annual benefit provided by substitution of the existing motor, given by:

$$\bar{AB} = H \cdot TE \cdot \bar{P_t} \cdot \left( \frac{1}{\eta_{old}} - \frac{1}{\eta_{new}} \right) \quad (15)$$

The calculation procedure employs the Monte Carlo Method, performing several simulations in which, there is an assortment of all variables in each one.

For each simulation, net benefit (NB) corresponding to the present net value for a given annual interest rate ($i$) is calculated.

$$NB = V_{F} - I_M + \sum_{t=1}^{n} \left( \frac{\bar{AB}}{(1 + i)^t} \right)$$

Although the annual benefits distributions are the same, the values assorted to each year do not need to be.

At the end of the simulations, the average and standard deviations are calculated for the group of values obtained for the net benefit, thus defining its distribution.

Opposing to engineering, the economical analysis admits risks larger than the ones often mentioned (5%). Values around 20% are usual and they will be employed here. Therefore, the only thing to do is to check which is the risk of the net benefit being negative and in case it is less than 20% the change of motors is economically interesting.
VI. EXAMPLE OF APPLICATION OF THE METHODOLOGY

A digital program was developed for the practical application of the technology described. In order to determine the working power of a substitute with its application, one of the twenty tested motors was used, whose characteristics are: rated working power (11 kW), nameplate rated speed (1740 rpm), measured working speed (1770 rpm), efficiency (0.70), variation in efficiency (0.04).

The errors assumed for the digital tachometer in the speed measurement were 0.1%.

The results are:

a) Working power:
   - average: 5.430 kW
   - deviation: 0.700 kW

b) chosen working power (risk=5%): 6.102 kW

Therefore, the working power determined for the substitute motor will be of 7.35 kW, because it is the closest commercial motor with working power higher then the chosen working power.

In order to check the economic feasibility for substitution, the following values are employed:

a) Operation time:
   - Expect: 4000 h;
   - Lower: 3600 h;
   - Upper: 4200 h;

b) Energy Tariff:
   - Expect: 50 US$/MWh;
   - Lower: 40 US$/MWh;
   - Upper: 60 US$/MWh;

c) New motor efficiency: 78%;
d) New motor investment:
   - Expect: US$ 250.00;
   - Lower: US$ 220.00;
   - Upper: US$ 270.00;
e) Residual value for the old motor:
   - Expect: US$ 60.00;
   - Lower: US$ 37.00;
   - Upper: US$ 63.00;
f) Usefull life for the new motor:
   - Expect: 19.4 years;
   - Lower: 16.0 years;
   - Upper: 20.0 years;
g) Annual interest rate: 12 % per year

With these values, the risk in a eventual replacement is less than 20%, showing the feasibility of the substitution of this 11 kW motor to another one of 7.35 kW.

VII. CONCLUSIONS

The method carried out in this paper has been proven worthwhile. Its practicability in an industrial environment is the main quality, specially when energy conservation is looked for.

Some improvements should be done, as the working torque estimate through current measurement. This can spread its use.

VIII. REFERENCES

PARAMETERS IDENTIFICATION OF A LOAD-MOTOR SET FOR ENERGY CONSERVATION: DYNAMIC ANALYSIS

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Abstract - In this paper, two methodologies for parameters identification of the drive set (motor and load) under a dynamic approach are presented. Simple measurements and low computer efforts are required. Thus, extended energy conservation studies become easy and fast. Simulation and least-square methods are employed to identify the parameters based on starting and stopping curves acquired. A comparison between the two proposed methods is pointed out. A real study case is carried out aiming to identify the parameters of a motor-fan set.

I. INTRODUCTION

The energy conservation has been shown as an interesting option to guarantee the supply of electrical power, either under the technical or the economical viewpoint.

By analyzing mainly the industrial sector, it is observed that the large consumption of electrical power is due to the drives employing three-phase induction motors that, in more than 70% of the capacity installed, move fluids through pumps, compressors and fans. It is also noted in the industrial environment that most motors operate with a loading under 80% of the rated capacity, implying in low efficiencies and low power factors.

The recommendation is to determine, as accurately as possible, the real load on the motor shaft and, in case of oversizing, to assess its substitution for another motor with more suitable characteristics for the drive. Under the energy conservation viewpoint this entails evident benefits. It is important to bear in mind that, in an industry, the number of motors is very high and that the measurement and test conditions are not always suited for good evaluation. Aiming quickness and simplicity of the analysis, [1] presents a method to estimate the load torque and selection of a substitute motor with a pre-determined risk being respected and analyzing only a static operating condition. In case of an eventual substitution, two questions can be risen:

- Will the new motor withstand the start-up and intermittent operations?
- Where to use the old motor?

The first question enhances the necessity of knowing the dynamic behavior of the load, which will permit the correct specification of the new motor and its protection adjustment. The second question is associated to the necessity of using the old motor somewhere else, helping to make its substitution economically feasible. In both cases, the evaluations in field are only necessary when there are no characteristics and reliable data available.

In this paper two methodologies are presented, which can be used in separately or jointly, for evaluation of the load and motor characteristics in an industrial environment. Differing from other works [2], there is no excessive concern with the model precision, but with the ease of application and achievement of results within an acceptable and trustable range. Another point that makes this method different from others is that the tests are performed over the load-motor set and not over the load and motor separately.

II. LOAD AND MOTOR MODELS

According to what has been exposed, there is no search for models of a refined accuracy. In this case, a simplified model capable of serving a thermal analysis of the motor is enough. For this, the following model can be adopted to represent the mechanical load:

\[ T_L = a \cdot \omega^b + T_S \]  

where:

- \( T_L \) - load torque (Nm);
- \( a, b, T_S \) - characteristic parameters;
- \( \omega \) - angular speed (rad/s).

It should be noted that this model is very adequate for turbomachines and, in most cases, \( b \) assumes a value quite close to 2 or 2 indeed. For the stopped machine (\( \omega = 0 \)), it is concluded from (1) that \( T_L \) equals \( T_S \). However, there is a singularity in the torque behavior between the the periods immediately before and after the motor starting, because the static friction is much higher than the dynamic friction. Therefore, the load resistant torque at start-up will be much higher than \( T_S \).
The dynamic behaviour of the system will be given by:

\[ T_M - T_L = I \cdot \frac{d\omega}{dt} \]  

(2)

where:

- \( T_M \) - motor torque (Nm);
- \( I \) - motor-load set inertia (kgm²).

It is possible to calculate \( I \), according to a classic proposition, based on (2). At the exact moment after switching-off the motor \( (t_{off}) \), \( T_M \) equals zero, so:

\[ I = \frac{T_L(t_{off})}{\frac{d\omega}{dt}|_{t=t_{off}}} \]  

(3)

The value of \( T_L(t_{off}) \) can be considered the same as the torque supplied by the motor at the moment of switching-off, that is:

\[ T_L(t_{off}) = T_M(t_{off}) \]  

(4)

Based on [1], \( T_M(t_{off}) \) can be obtained by:

\[ T_M(t_{off}) = \frac{\omega_S - \omega(t_{off})}{\omega_S - \omega_R} \cdot \frac{P_R}{\omega_R} \]  

(5)

where:

- \( P_R \) - rated power (kW)
- \( \omega_R \) - rated speed (rad/s)
- \( \omega_S \) - synchronous speed (rad/s)

With the value of the speed at the moment of the switching-off, \( \omega(t_{off}) \), the motor torque can be calculated.

In this work, the following simplified model will be used for the motor:

\[ T_M = \frac{s}{\alpha \cdot s^2 + \beta \cdot s + \gamma} \]  

(6)

where \( \alpha, \beta, \gamma \) are parameters inherent to each motor and \( s \) is the slip for a given speed \( (\omega) \), calculated by:

\[ s = \frac{\omega_S - \omega}{\omega_S} \]  

(7)

This model does include neither the skin effect on the deep rotor bars nor the saturation effect presented during the start-up process. A second approximation that incorporates these effects is given by:

\[ T_M = \frac{s}{\alpha \cdot s^2 + \beta \cdot s + \gamma} + \frac{s}{\phi \cdot s^2 + \mu \cdot s + \theta} \]  

(8)

The behaviour outlined in (8) is the result of the summation of two characteristic curves, as shown in Fig. 1.

![Resulting torque characteristic](image_url)

Fig. 1. Resulting torque characteristic.

It should be noted that for high speeds, the second portion of (8) starts becoming neglectable, falling over (6), and vice-versa. So, as it was developed in (3), it is possible to obtain the motor start-up torque, \( T_M(t_{off}) \), based on the same speed register:

\[ T_M(t_{off}) = I \cdot \frac{d\omega}{dt}|_{\omega = \omega_S} \]  

(9)

However, it is to be enhanced the difficulty in obtaining a good measurement of the derivative in the start-up region [2]. Many times, such an evaluation is not consistent to the reality.

Finally, in all models proposed here, the nameplate data are assumed to be true. Effects of variation of the feeding voltage, caused by start-up methods, over the motor developed torque, can be incorporated to the models considering the torque proportional to the square of the voltage applied.

### III. DATA ACQUISITION

Contrary to the traditional methods employed for the identification of three-phase induction motor parameters, where digital or analog precision systems are required [3], the tools proposed here are easy to be applied in the field and the measurements can be performed even with electromechanical recorders.

What is required in this case is the records of the start-up speed of the load-motor set, until its stabilization, and of the
topping speed, until it is almost fully stopped. The knowledge about the behaviour of the feeding voltage is also necessary, permitting the correction of the effects of its variation, either caused by feeder voltage drop or by some start-up method, over the motor torque.

Fig. 2 presents a typical recording of the speed during the start-up, normal operation and stopping of a motor-load set.

![Graph: Speed vs Time](image)

**Fig. 2.** Speed behavior in start-up, normal operation and stopping of a motor-load set.

In Fig. 2, \( t_{on} \) is the start-up instant, \( t_v \) is an instant in which a voltage variation takes place (for example, switching \( Y/\Delta \)) and \( t_{off} \) is the switching-off instant.

The points of the region between \( t_{on} \) and \( t_v \) are very important for the calculation of the relative parameters to the skin and saturation effects. Between \( t_2 \) and \( t_3 \) are located the most important points for determining of \( \alpha, \beta \) and \( \gamma \). The inertia is defined exclusively by the data contained between \( t_{off} \) and \( t_5 \). From \( t_4 \) to \( t_5 \) are the points that influence the calculation of \( \alpha \) and \( \beta \). Finally, \( T_5 \) depends upon the data recorded from \( t_3 \), although it is very influenced by the section \( t_4 \) and \( t_5 \).

### IV. IDENTIFICATION TECHNIQUES

All the techniques presented here are exemplified based on a motor-centrifugal fan set with reduced voltage start-up (\( Y/\Delta \)). The motor data is as follows:

- rated power (\( P_r \)) : 36.8 (kW)
- rated speed (\( \omega_r \)) : 1760 (rpm)
- rated current (\( I_r \)) : 120/70 (A)
- rated voltage (\( P_V \)) : 220/380 (V)

Based on the measured working speed (1791 rpm), the torque (\( T_L \)) is evaluated.

\[ T_L = 44.89 \text{ (Nm)} \]

Fig. 3 shows the start-up recording (with reduced voltage), normal operation (with the voltage increased) and the decelerating of the set, done with an electromechanical recorder.

![Graph: Speed vs Time](image)

**Fig. 3.** Start-up recording, normal operation and stopping of the motor-fan set.

#### A. Calculating the Inertia

Both in the simulation method as in the least squares method it is possible to determine the inertia (1) straight from the proposed process. However, as seen in [3], this parameter is associated to the speed derivative, being subject to large variations no matter how small the changes in the \( \omega x t \) are.

In this way, it is recommended the previous calculation of the inertia, without concerning the identification technique used. For this, the classical method can be used. The calculation of the speed derivative can be performed by two processes: the graphical one, with low precision, and the regression one. The application of the linear regression technique over the points of the "\( \omega x t \)" curve should be avoided, because this process will be subject to the error often mentioned, due to the sensitivity of the derivative concerning to the curve chosen.

The recommendation is to search a function that adjusts itself to the points of the "\( \Delta \omega/\Delta t x t \)" curve instead of "\( \omega x t \)". The approximation to the \( \Delta \omega/\Delta t \) derivative should be calculated for intervals immediately after \( t_{off} \). The values calculated will be associated to the time located at the center of the respective interval. Over these points, the least squares technique is applied. The extrapolation of the curve obtained for time \( t_{off} \) results in the derivative of interest, which will serve to the calculation of the inertia using (3).
This technique avoids the visual errors of the graphical method and the sensitivity to the choice of the function in the case of the regression “$\omega x t$”. The regression over the points “$\Delta \omega / \Delta t \times t^n$” works as a filter, precluding sudden variations of the derivative.

Fig. 4 presents the results obtained for the above example.

As a conclusion, $R^2$ should be used as a choice criteria of the adjusted parameters, provided that a visual analysis admits the curve chosen as being a good estimation of the real curve.

Based on equations (1), (2) and (6), the block diagrams can be built for start-up and stopping of the set, as shown in Fig. (5) and (6), respectively.

Fig. 5. Block diagrams for the stopping of the set.

Fig. 6. Block diagrams for the start-up of the set.

The practicability of the simulation method depends on the initial estimates of the parameters. If such estimates are not good, the curve simulated may have no meaning, or numerical problems in the simulation may even show up.

From the rated, start-up and maximum conditions, very good estimates can be made for $a$, $b$, $c$, and $d$, and from them, to perform a refining. From the rated condition:

$$T = \frac{Pr \cdot sR}{R \cdot \omega R \cdot a \cdot sR + b \cdot \omega R + c}$$

(10)
At the start-up, \( s \) equals one and the start-up torque \( (T_{ON}) \) will be given by:

\[
T_{ON} = \frac{1}{\alpha + \beta + \gamma}
\]  

(11)

The maximum torque \( (T_{MAX}) \) is associated to a slip \( s_{MAX} \), which can be calculated taken the derivate of (10) for the variable \( s \) and equating it to zero.

\[
s_{MAX} = \sqrt{\gamma / \alpha}
\]  

(12)

\[
T_{MAX} = \frac{s_{MAX}}{\alpha + \beta + \gamma}
\]  

(13)

With equations (10), (11), (12) and (13) in hands, it is possible to calculate \( \alpha \), \( \beta \) and \( \gamma \) from estimates for \( T_{ON} \), \( T_{MAX} \) and \( s_{R} \). In the above example, the following values are adopted:

\[
\begin{align*}
T_{R} &= P_{R}/\omega_{R} \\
s_{R} &= (\omega_{S} - \omega_{R})/\omega_{S} \\
T_{ON} &= T_{R} \\
T_{MAX} &= 2T_{R} \\
s_{MAX} &= 2s_{R}
\end{align*}
\]

That results in the following estimates:

\[
\begin{align*}
\alpha &= 0.00823321 \\
\beta &= 0.00678586 \\
\gamma &= 0.00001626
\end{align*}
\]

For the load, \( b \) is assumed as 2, as reported previously. \( T_{S} \) can be assumed as a fraction of the working torque. Here 5\% was adopted. With \( b \) and \( T_{S} \) in hands, and based on the initial condition, \( a \) is calculated, as follows:

\[
a = \frac{T_{L}(t_{off}) - T_{S}}{\omega^{b}(t_{off})}
\]  

(14)

In the above example, the following values were achieved:

\[
\begin{align*}
a &= 0.000012 \\
b &= 2 \\
T_{S} &= 2.25
\end{align*}
\]

With the simulation models and the initial estimates in hands, the stopping simulation should be firstly proceeded, for load identification and, secondly, the start-up, where the motor will be identified. By performing several simulations for the stopping, the curve shown in the figure that was adopted as a good choice. The parameters that generated this curve are:

\[
\begin{align*}
a &= 0.0000845 \\
b &= 2.5 \\
T_{S} &= 4.2
\end{align*}
\]

Figure 7. Simulation for the stopping.

By adopting the simplified model for the motor, and the parameters already calculated from the load, the curve is simulated in Fig. 8, that was obtained with the following parameters after several simulations:

\[
\begin{align*}
a &= 0.0086667 \\
b &= 0.0033333 \\
\gamma &= 0.0000166
\end{align*}
\]

Fig. 8. Simulation for the start-up.
C. The Least Squares Method

Based on the assumptions adopted, the following model of motor start-up can be obtained:

\[-a \cdot \omega^b - T_s = I \cdot \frac{d\omega}{dt}\]  \hspace{1cm} (15)

In order to avoid a trivial solution, (15) is divided by the inertia and, considering \( b = 2 \):

\[\frac{d\omega}{dt} = \frac{a}{J} \cdot \omega(t)^2 - \frac{T_s}{J}\]  \hspace{1cm} (16)

In terms of linear regression, (16) can be written as follows:

\[y = -A_0 - A_1 \cdot x\]  \hspace{1cm} (17)

where:

\[y = \frac{d\omega}{dt}, \quad x = \omega(t)^2, \quad A_0 = \frac{a}{J}, \quad A_1 = \frac{T_s}{J}\]

This equation allows the use of the conventional least squares algorithm for its solution [5]. In this case, the values of \( A_0 \) and \( A_1 \) will be calculated. To return to the values of \( a \) and \( T_s \), the following initial conditions can be assumed:

\[\omega_0 = \omega(t_0), \quad T_0 = T_{0\text{eff}}\]

So:

\[\hat{a} = \frac{\omega_0^2}{\omega_0^2 + (A_0/A_1)}\]  \hspace{1cm} (18)

\[\hat{A}_1 = \frac{T_0}{\omega_0^2 + (A_0/A_1)}\]  \hspace{1cm} (19)

\[\hat{T}_s = \frac{T_0 \cdot A_0}{s \cdot \omega_0^2 + (A_0/A_1)}\]  \hspace{1cm} (20)

It is interesting to note that the inertia value can be identified or previously calculated using (3). In this case, the following expressions are valid:

\[\hat{a} = A_1 \cdot I\]  \hspace{1cm} (21)

\[\hat{T}_s = A_0 \cdot I\]  \hspace{1cm} (22)

This fact is particularly important when the value of \( b \) is not ascertained. Table I shows the large spread of the parameters identified in face of small variations of \( b \). The adoption of a fixed value for the inertia avoids this problem, giving a smaller freedom degree for the calculation of \( \hat{a} \) and \( \hat{T}_s \).

<table>
<thead>
<tr>
<th>( b )</th>
<th>( \hat{a} )</th>
<th>( I )</th>
<th>( \hat{T}_s )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.92</td>
<td>1.223 ( 10^{7} )</td>
<td>3.7105</td>
<td>1.8636</td>
<td>0.9952</td>
</tr>
<tr>
<td>1.96</td>
<td>1.207 ( 10^{7} )</td>
<td>4.5092</td>
<td>2.4229</td>
<td>0.9955</td>
</tr>
<tr>
<td>2.00</td>
<td>1.187 ( 10^{7} )</td>
<td>5.4604</td>
<td>3.1201</td>
<td>0.9958</td>
</tr>
<tr>
<td>2.04</td>
<td>1.163 ( 10^{7} )</td>
<td>6.5836</td>
<td>3.9801</td>
<td>0.9960</td>
</tr>
<tr>
<td>2.08</td>
<td>1.133 ( 10^{7} )</td>
<td>7.8964</td>
<td>5.0284</td>
<td>0.9961</td>
</tr>
</tbody>
</table>

Likewise, for the motor start-up, the following modelling is employed:

\[T_M - T_L = I \cdot \frac{d\omega}{dt}\]  \hspace{1cm} (23)

\[\frac{s}{\alpha \cdot s^2 + \beta \cdot s + \gamma} = \frac{\frac{d\omega}{dt}}{\frac{l \cdot (d\omega)}{dt} + a \cdot \omega^b + T_s}\]  \hspace{1cm} (24)

From it, one can obtain:

\[\alpha \cdot s^2 + \beta \cdot s + \gamma = \frac{s}{I \cdot (d\omega/dt) + a \cdot \omega^b + T_s}\]  \hspace{1cm} (25)

The value of \( \gamma \) can be calculated for the motor rated conditions:

\[\gamma = (s_R/T_R) - \alpha \cdot s_R^2 - \beta \cdot s_R\]  \hspace{1cm} (26)

By applying (26) in (25):

\[\alpha \cdot (s^2 - s_R^2) + \beta \cdot (s - s_R) = \frac{s \cdot (U/ U_R)^2}{I \cdot (d\omega/dt) + a \cdot \omega^b + T_s} \cdot \frac{s_R}{T_R}\]  \hspace{1cm} (27)

This is the model of linear regression for determining the parameters \( \alpha \) and \( \beta \) of the motor. It should be noted that this expression considers the variation of the motor torque with respect to the voltage applied during its start-up. The parameter \( \gamma \) is given by expression (26).

Applying this methodology to the stopping data, the following results are obtained:

\[I = 5.4604\]
\[a = 0.001875\]
\[b = 2\]
\[T_s = 3.1202\]
Performing a stopping simulation with these parameters yields the curve shown in the Fig. 9.

![Fig. 9. Result of the stopping simulation.](image)

For the starting data, one can obtain with the proposed methodology, the following motor parameters.

\[
\alpha = 0.0083871 \\
\beta = 0.0029039 \\
\gamma = 0.0000427
\]

Fig. 10 shows the result of the starting simulation using the above parameters.

![Fig. 10. Simulation of the starting process.](image)

The motor torque characteristic is presented in the Fig. 11, using the calculated \( \alpha \), \( \beta \) and \( \gamma \).

![Fig. 11. Motor torque characteristic.](image)

V. MOTOR SUBSTITUTION

Aiming energy conservation, the motor can be substituted by a lower power one. That new motor should have appropriate characteristics to be able to bear the start up and normal operation.

In this case, the chosen motor has the following main data and its torque vs. speed curve is present in Fig. 12.

\[
P_R = 11 \text{ (kW)} \\
\omega_R = 1763 \text{ (rpm)} \\
T_{ON} = 100 \text{ (Nm)} \\
T_{MAX} = 200 \text{ (Nm)} \\
s_{MAX} = 0.19
\]

One highlights the high start up torque that allows a fast acceleration. \( \alpha \), \( \beta \) and \( \gamma \) can be calculated based on the above data.

Regarding to the current of the new motor, a full voltage starting is possible.

Thus, the starting simulation curve is obtained. This curve is in Fig. 13.
VI. CONCLUSION

The two presented techniques for parameters identification of motor-load set—simulation and least-squares—show to be useful and precise. However, it is necessary to highlight that the simulation methodology spends much more time than least-squares one. This additional efforts is caused by the high number of simulations siming a fine tuning of the parameters.

Beside, the simulation provides a physical feeling to the technician, reducing risks to get values with no sense.

If a bad choice of the initial values is done, the simulations can reach problems, as instability, or high number of trials to adjust the parameters.

The least-squares methodology is too fast and, on can say, “quasi-automatic”. It means that the final result is obtained with intermediate analysis. This quality is also a disadvantage, because one can get values without physical sense. So always it is necessary to simulate the behaviour of the set, incorporating the estimated parameters, to verify the results.

The best is a combination of the methods: one firstly calculates the parameters by least-squares and, finally, looks for a fine tuning by simulation, incorporating the advantages of both methodologies.

At last, the methods were confirmed to be robusts, even when the data were obtained from rough methodologies, as usual acquisition of time and speed.

VII. REFERENCES